Trisect a Line Segment (4)

Method:

We will construct a Centroid and prove trisection of a segment using Centroid.

1) Construct a segment AB
2) Draw a perpendicular line to segment AB through point B
3) Take an arbitrary point C on the perpendicular line
4) Construct a segment BC
5) Then construct segment BD such that BC = BD (*hence AB is bisector of segment CD*)
6) Construct midpoints of AC and AD and label them E and F respectively
7) Construct segment DE and CF (*hence the intersection of 3 bisectors AB, DE, and CF constructs a Centroid at point G)*
8) Construct segment BG
9) Construct segment GH such that BG = GH.

Claim: So points G and H trisects the segment AB

Proof:

The proof is based on the properties of a Centroid. First, a *Median* is the segment connecting the a vertex of a triangle to its opposite side. So AB, CF, and DE are medians of ΔACD. Furthermore, a *Centroid* is the point where three medians of a triangle intersect. Hence point G is the Centroid of ΔACD.

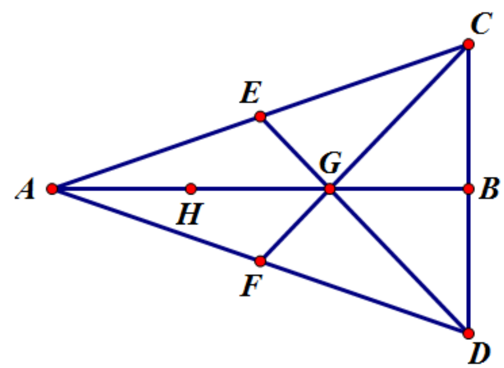

Figure 1: Construction of a Centroid

Now, Lets construct segment EF(Midsegment of the triangle). By construction, $EF \parallel CD$ and $EF = \frac{1}{2}CD$. So, considering triangles CGD and EGF, $\angle EGF \cong \angle CGD$, since they are vertical angles. Similarly, $\angle GEF \cong \angle GCD$ and $\angle EFG \cong \angle GDC$, since they are alternate interior angles of parallel lines EF and CD. So, due to Angle-Angle-Angle relationship, $\Delta EGF \cong \Delta CDG$.

Since $\Delta EGF : \Delta CDG = 1:2$. Hence we have

$$EF : CD = 1:2; EG : GD = 1:2; FG : GC = 1:2$$

In other words, point G (the Centroid) is $\frac{2}{3}$ of the distance from A to B, C to F, and E to D.

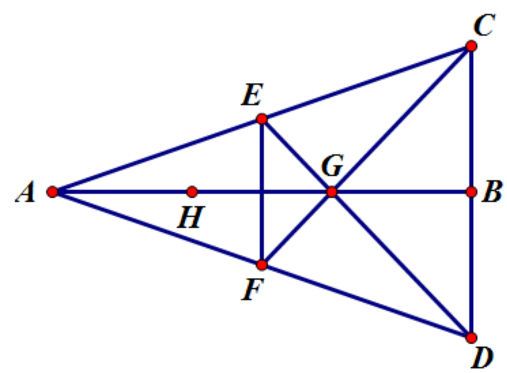

Figure 2: Proof of Trisect of Segment AB

So, BG = $\frac{1}{3}$AB. Now create another segment GH such that GH = BG. So points G and H trisect the segment AB.